

ACTIVITIES

ACTIVITY 1

AIM

To make a paper scale of given least count: (a) 0.2 cm and (b) 0.5 cm

APPARATUS AND MATERIAL REQUIRED

Thick ivory/drawing sheet; white paper sheet; pencil; sharpener; eraser; metre scale (ruler); fine tipped black ink or gel pen.

PRINCIPLE

Least count of a measuring instrument is the smallest measurement that can be made accurately with the given measuring instrument. A metre scale normally has graduations at 1 mm (or 0.1 cm) spacing, as the smallest division on its scale. You cannot measure lengths with this scale with accuracy better than 1mm (or 0.1 cm).

You can make paper scale of least count (a) 0.2 cm (b) 0.5 cm, by dividing one centimetre length into smaller divisions by a simple method, without using mm marks.

PROCEDURE

(a) Making Paper Scale of Least Count 0.2 cm

1. Fold a white paper sheet in the middle along its length.
2. Using a sharp pencil, draw a line AB, of length 30 cm in either half of the white paper sheet [Fig. A1.1(a)].
3. Starting with the left end marked A as zero, mark very small dots on the line AB after every 1.0 cm and write 0, 1, 2 ..., 30 at successive dots.
4. Draw thin, sharp straight lines, each 5 cm in length, perpendicular to the line AB at the position of each dot mark.
5. Draw 5 thin, sharp lines parallel to the line AB at distances of 1.0 cm, 2.0 cm, 3.0 cm, 4.0 cm and 5.0 cm respectively. Let the line at 5 cm be DC while those at 1 cm,

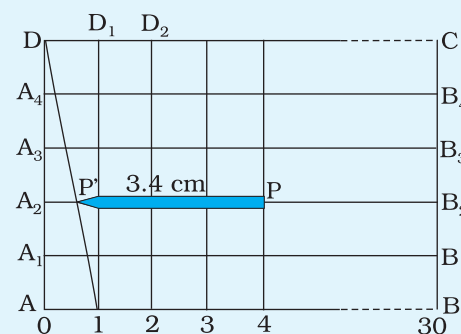


Fig. A1.1(a): Making a paper scale of least count 0.2 cm

2 cm, 3 cm, and 4 cm be A_1B_1 , A_2B_2 , A_3B_3 and A_4B_4 respectively [Fig A 1.1(a)].

- Join point D with the dot at 1 cm on line AB. Intersection of this line with lines parallel to AB at A_4 , A_3 , A_2 and A_1 are respectively 0.2 cm, 0.4 cm, 0.6 cm and 0.8 cm in length.
- Use this arrangement to measure length of a pencil or a knitting needle with least count of 0.2 cm.

(b) Making Paper Scale of Least Count 0.5 cm

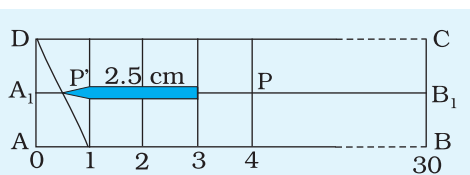


Fig. A1.1(b):

- Using a sharp pencil, draw a line AB of length 30 cm in the other half of the white paper sheet [Fig. A1.1(b)].
- Repeat steps 3 to 6 as in the above Activity 1.1(a), but draw only two lines parallel to AB at distances 1.0 cm and 2.0 cm instead of 5 cm.
- Join diagonal 1-D by fine tipped black ink pen [Fig. A 1.1 (b)].

- Use this scale to measure length of a pencil/knitting needle with least count of 0.5 cm. Fractional part of length 0.5 cm is measured on line A_1B_1 .

(c) Measuring the Length of a Pencil Using the Paper Scales A and B

- Place the pencil PP' along the length of the paper scale A (least count 0.2 cm) such that its end P is on a full mark (say 1.0 cm or 2.0 cm etc. mark). The position of the other end P' is on diagonal 1-D. If P' goes beyond the diagonal, place it on next upper line, in which fraction of intersection is 0.2 cm larger, and so on. Thus, in Fig. A 1.1 (a), length of the pencil = 3 cm + $.2 \times 2$ cm = 3.4 cm. Take care that you take the reading with one eye closed and the other eye directly over the required graduation mark. The reading is likely to be incorrect if the eye is inclined to the graduation mark.
- Repeat preceding step 1, using the paper scale B having least count 0.5 cm and record your observation in proper units.

OBSERVATIONS

Least count of the paper scale **A** = 0.2 cm

Least count of the paper scale **B** = 0.5 cm

RESULT

- Scale of least count 0.2 cm and 0.5 cm have been made; and
- Length of pencil as measured by using the scales made above is
(a) ... cm and (b) ... cm.

PRECAUTIONS

- (i) Very sharp pencil should be used.
- (ii) Scale should be cut along the boundary by using a sharp paper cutter.
- (iii) Observation should be recorded showing accuracy of the scale.
- (iv) While measuring lengths, full cm mark should be made to coincide with one end of the object and other end should be read on the scale.

SOURCES OF ERROR

The line showing the graduations may not be as sharp as required.

DISCUSSION

1. The accuracy of measurement of length with the scale so formed depends upon the accuracy of the graduation and thickness of line drawn.
2. Some personal error is likely to be involved e.g. parallax error.

ACTIVITY 2

AIM

To determine the mass of a given body using a metre scale by the principle of moments.

APPARATUS AND MATERIAL REQUIRED

A wooden metre scale of uniform thickness (a wooden strip of one metre length having uniform thickness and width can also be used); load of unknown mass, wooden or metal wedge with sharp edge, weight box, thread (nearly 30 cm long), a spirit level, and a raised platform of about 20 cm height (such as a wooden or metal block).

PRINCIPLE

For a body free to rotate about a fixed axis, in equilibrium, the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

If M_1 is the known mass, suspended at a distance l_1 on one side from the centre of gravity of a beam and M_2 is the unknown mass, suspended at a distance l_2 on the other side from the centre of gravity, and the beam is in equilibrium, then $M_2 l_2 = M_1 l_1$.

PROCEDURE

1. Make a raised platform on a table. One can use a wooden or a metal block to do so. However, the platform should be a sturdy, place a wedge having a sharp edge on it. Alternately one can fix the wedge to a laboratory stand at about 20 cm above the table top. With the help of a spirit level set the level of the wedge horizontal.
2. Make two loops of thread to be used for suspending the unknown mass and the weights from the metre scale (beam). Insert the loops at about 10 cm from the edge of the metre scale from both sides.
3. Place the metre scale with thread loops on the wedge and adjust it till it is balanced. Mark two points on the scale above the wedge where the scale is balanced. Join these two points with a straight line which would facilitate to pin point the location of balance

position even if the scale topples over from the wedge due to some reason. This line is passing through the centre of gravity of scale.

- Take the unknown mass in one hand. Select a weight from the weight box which feels nearly equal to the unknown mass when it is kept on the other hand.
- Suspend the unknown mass from either of the two loops of thread attached to the metre scale. Suspend the known weight from the other loop (Fig. A 2.1).
- Adjust the position of the known weight by moving the loop till the metre scale gets balanced on the sharp wedge. Make sure that in balanced position the line drawn in Step 3 is exactly above the wedge and also that the thread of two loops passing over the scale is parallel to this line.
- Measure the distance of the position of the loops from the line drawn in Step 3. Record your observations.
- Repeat the activity atleast two times with a slightly lighter and a heavier weight. Note the distances of unknown mass and weight from line drawn in Step 3 in each case.

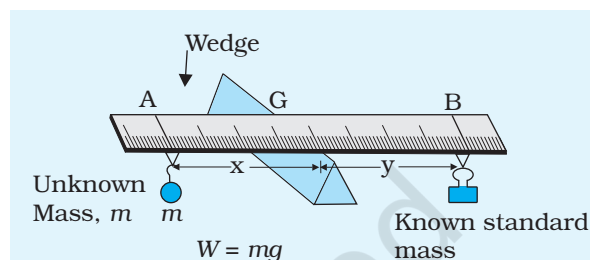


Fig. A 2.1: Experimental set up for determination of mass of a given body

OBSERVATIONS

Position of centre of gravity = ... cm

Table A 2.1: Determination of mass of unknown object

S. No.	Mass M_1 suspended from the thread loop to balance the metre scale (g)	Distance of the mass from the wedge l_1 (cm)	Distance of solid of unknown mass from the wedge l_2 (cm)	Mass of unknown load M_2 (g) $= \frac{M_1 l_1}{l_2}$	Average mass of unknown load (g)
1					
2					
3					
4					
5					

CALCULATIONS

In balanced position of the metre scale, moment of the force on one side of the wedge will be equal to the moment of the force on the other side.

Moment of the force due to known weight = $(M_1 l_1) g$

Moment of the force due to unknown weight = $(M_2 l_2) g$

In balanced position

$$M_1 l_1 = M_2 l_2$$

or
$$M_2 = \frac{M_1 l_1}{l_2}$$

Average mass of unknown load = ... g

RESULT

Mass of given body = ... g (within experimental)

PRECAUTIONS

1. Wedge should be sharp and always perpendicular to the length of the scale.
2. Thread loops should be perpendicular to the length of the scale.
3. Thread used for loops should be thin, light and strong.
4. Air currents should be minimised.

SOURCES OF ERROR

1. Mass per unit length may not be uniform along the length of the metre scale due to variation in its thickness and width.
2. The line marked on the scale may not be exactly over the wedge while balancing the weights in subsequent settings.
3. The thread of the loops may not be parallel to the wedge when the weights are balanced, which in turn would introduce some error in measurement of weight-arm.
4. It may be difficult to adjudge balance position of the scale exactly. A tilt of even of the order of 1° may affect the measurement of mass of the load.

DISCUSSION

1. What is the name given to the point on the scale at which it is balanced horizontally on the wedge?
2. How does the least count of the metre scale limit the accuracy in the measurement of mass?
3. What is the resultant torque on the metre scale, due to gravitational force, when the scale is perfectly horizontal?
4. Explain, how a physical balance works on the principle of moments.
5. What problems would air currents cause in this activity?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. We can determine the accuracy of various weights available in the laboratory, by finding out their mass by the above method and comparing with their marked values.
2. Verify the principle of moments using a metre scale. After balancing the metre scale at its centre of gravity, suspend masses M_1 and M_2 at distances l_1 and l_2 respectively, from the centre of gravity, on either side. Adjust the distances l_1 and l_2 so that the metre scale is horizontal. Calculate and compare $M_1 l_1$ and $M_2 l_2$. Repeat with other combinations of masses M_1 and M_2 .

ACTIVITY 3

AIM

To plot a graph for a given set of data choosing proper scale and show error bars due to the precision of the instruments.

APPARATUS AND MATERIAL REQUIRED

Graph paper, a pencil, a scale and a set of data

PRINCIPLE

Graphical representation of experimentally obtained data helps in interpreting, communicating and understanding the interdependence between the variable parameters of a given phenomena. Measured values of variables have some error or expected uncertainty. For this reason each data point on the graph cannot have a unique position. That means depending upon the errors, the x-axis coordinate and y-axis coordinate of every point plotted on the graph will lie in a range known as an error bar.

Any measurement using a device has an uncertainty in its value depending on the precision of the device used. For example, in the measurement of diameter of a spherical bob, the correct way is to represent it $d \pm \Delta d$, where Δd is the uncertainty in measurement of d given by the least count of the vernier/screw gauge used. Representation of $d \pm \Delta d$ in a graph is shown as a line having a length of $\pm \Delta d$ about point 'd'. This is known as the error bar of d .

We take an example where the diameters of objects, circular in shape, are measured using a vernier calipers of least count 0.01 cm. These measured values are given in Table 1. From the measured values of diameters, it is required to calculate the radius of each object and to round off the digits in the radius to the value consistent with the least count of the measuring instrument, in this case, the vernier calipers. We also estimate the maximum possible fractional uncertainty (or error) in the values of radius. Next, the area A of each object is then calculated using the formula.

$$\text{Area, } A = \frac{\pi d^2}{4}$$

where π is the well-known constant.

Graphical representation of experimental data provides a convenient way to look for interdependence or patterns between various parameters associated with a given experiment or phenomenon or an event. Graphs also provide a useful tool to communicate a given data in pictorial form. We are often required to graphically represent the data collected during an experiment in the laboratory, to verify a given relation or to infer inter-relationships between the variables. It is, therefore, imperative that we must know the method for representing a given set of data on a graph, develop skill to draw most appropriate curve to represent the plotted data and learn as to how to interpret a given graph to infer relevant information.

Basic ideas about the steps involved in plotting a line graph for a given data and finding the slope of the curve have already been discussed in Chapter I. The steps involved in plotting a graph include choice of axes (independent variable versus dependent variable), choice of scale, marking the points on the graph for each pair of data and drawing a smooth curve/line by joining maximum number of points corresponding to the given data. Interpretation of the graph usually involves finding the slope of the curve/line, inferring nature of dependence between variables/parameters, interpolating/extrapolating the graph to find desired value of the dependable variable corresponding to a given value of independent variable or vice versa. However, so far you have learnt to graphically represent the data for which uncertainty or error is either ignored or is presumed not to exist. As you know every data has some uncertainty/error due lack of precision in measurement or some other factors inherent in the process/method of data collection. It is possible to plot a graph that depicts the extent of uncertainty/error in the given data. Such a depiction in the graph is called an **error bar**. In general error bars allow us to graphically illustrate actual errors, the statistical probability of errors in the measurement or typical data points in comparison to the rest of the data.

You have learnt to show uncertainty in measurement of a physical quantity like length, mass, temperature and time on the basis of the least count of the measuring instruments used. For example, the diameter of a wire measured with a screw gauge having least count 0.001 cm is expressed as 0.181 cm \pm 0.001 cm. The figure \pm 0.001 cm in the measurement indicates that the actual value of diameter of the wire may lie between 0.180 and 0.182 cm. However, the error in measurement may also be due to many other factors, such as personal error, experimental error etc. In some cases the error in data may be due to factors other than those associated with measurement. For example, angles of scattering of charge particles in an experiment on scattering of α -particles or opinion collected from a section of a population on a social issue. The uncertainty due to such errors is estimated through a variety of statistical methods about which you will learn in higher classes. Here we shall consider uncertainty in

measurements only due to the least count of the measuring instrument so as to learn how uncertainty for a given data is shown in a line graph.

Let us take the example of the graph between time period, T , and the length, l , of a simple pendulum. The uncertainty in measurement of time period will depend on the least count of the stop watch/clock while that in measurement of length of the pendulum will depend on the least count of the device(s) used to measure length. Table A 3.1 gives the data for the time period of simple pendulum measured in an experiment along with the uncertainty in measurement of the length and time period of the pendulum.

Table A 3.1 Time period of simple pendulums of different lengths

S. No.	Length of the pendulum		Time period		
	Length as measured with metre scale, L (cm)	Length with uncertainty in L (least count of scale 0.1 cm) (cm)	Average time period as measured with stop watch, T (s)	Time period with uncertainty in T (least count of stop watch 0.1 s) (s)	Square of time period T^2 with uncertainty
1	80.0	80 ± 0.1	1.8	1.8 ± 0.1	3.24 ± 0.2
2	90.0	90 ± 0.1	1.9	1.9 ± 0.1	3.61 ± 0.2
3	100.0	100 ± 0.1	2.0	2.0 ± 0.1	4.0 ± 0.2
4	110.0	110 ± 0.1	2.1	2.1 ± 0.1	4.41 ± 0.2
5	120.0	120 ± 0.1	2.2	2.2 ± 0.1	4.84 ± 0.2
6	130.0	130 ± 0.1	2.3	2.3 ± 0.1	5.29 ± 0.2
7	140.0	140 ± 0.1	2.4	2.4 ± 0.1	5.76 ± 0.2
8	150.0	150 ± 0.1	2.4	2.4 ± 0.1	5.76 ± 0.2

P LOTTING OF A GRAPH WITH ERROR BARS

Steps involved in drawing a graph with error bars on it are as follows:

1. Draw x- and y- axes on a graph sheet and select an appropriate scale for plotting of the graph. In order to show uncertainty/error in given data, it is advisable that the scale chosen should be such that the lowest value of uncertainty/error on either axes could be shown by at least the smallest division on the graph sheet.
2. Mark the points on the graph for each pair of data without taking into account the given uncertainty/error.

3. Each point marked on the graph in Step 2 has an uncertainty in the value shown on either the x-axis or the y-axis or both. For

example, let us consider the case for the point corresponding to (80, 1.8) marked on the graph. If we take into account the uncertainty in measurement for this case, the actual length of the pendulum may lie between 79.9 cm and 80.1 cm. This uncertainty in the data is shown in the graph by a line of length 0.2 cm drawn parallel to x-axis with its midpoint at 80.0 cm, in accordance with the scale chosen. The line of length 0.2 cm parallel to x-axis shows the error bar for the pendulum of length 80.0 cm. One can similarly draw error bar for each length of the pendulum.

4. Repeat the procedure explained in Step 3 to draw error bars for uncertainty in measurement of time period. However, the error bars in this case will be parallel to the y-axis.
5. Once the error bars showing the uncertainty for data in both the axes of the graph have been marked, each pair of data on the graph will be marked with a + or < or > sign, depending on the extent of uncertainty and the scale chosen for each axis, instead of a point usually marked for drawing line graph (Fig. A 3.1).
6. A smooth curve drawn passing as close as possible through all the + marks marked on the graph, instead of points, gives us the plot between the two given variables (Fig. A 3.2).

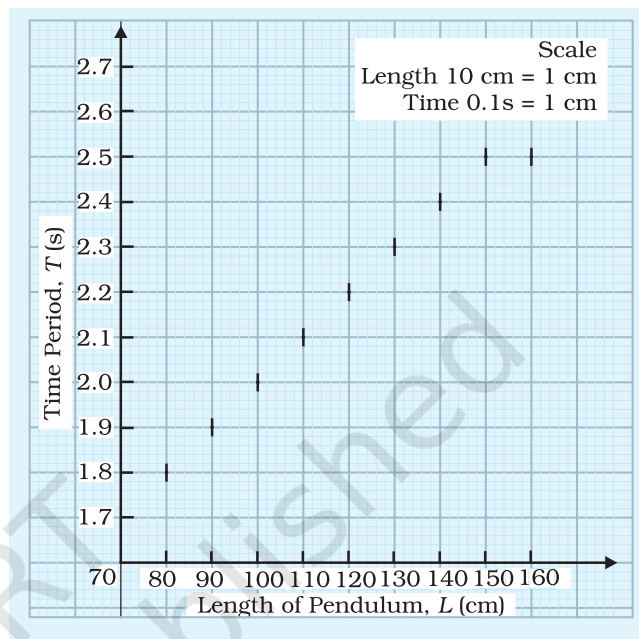


Fig. A 3.1: Error bars corresponding to uncertainty in time period of the given pendulum (uncertainty in length is not shown due to limitation of scale)

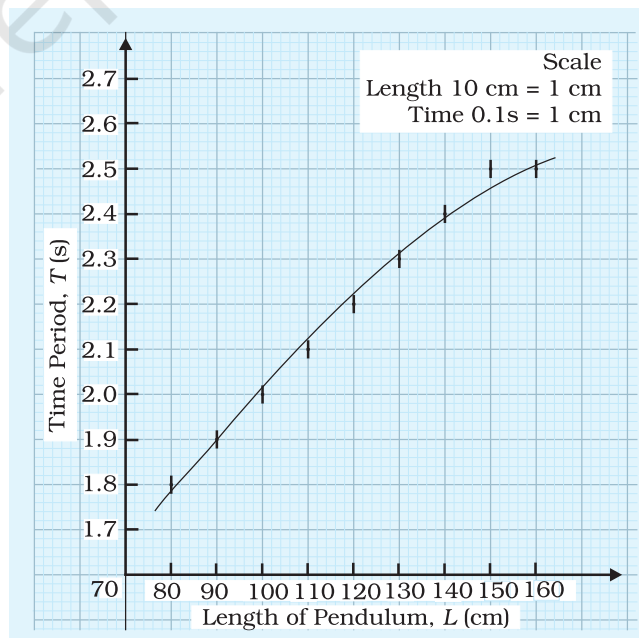


Fig. A 3.2: Graph showing variation in time period of a simple pendulum with its length along with error bars

RESULT

A given set of data gives unique points. However, when plotted, a curve representing that data may not physically pass through these points. It must, however, pass through the area enclosed by the error bars around each point.

PRECAUTIONS

1. In this particular case the point of intersection of the two x-axis and y-axis represent the origin of O at (0, 0). However, this is not always necessary to take the values of physical quantities being plotted as zero at the intersection of the x-axis and y-axis. For a given set of data, try to maximize the use of the graph paper area.
2. While deciding on scale for plotting the graph, efforts should be made to choose a scale which would enable to depict uncertainty by at least one smallest division on the graph sheet.
3. While joining the data points on the graph sheet, enough care should be taken to join them smoothly. The curve or line should be thin.
4. Every graph must be given a suitable heading, which should be written on top of the graph.

SOURCES OF ERROR

1. Improper choice of origin and the scale.
2. Improper marking of observation points.

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

how error bars in the graphs plotted for the data obtained while doing Experiment Nos. 6, 9, 10, 11, 14 and 15.

Note:

As the aim of the Activity is to choose proper scale while plotting a graph alongwith uncertainty only due to the measuring devices, the calculation in the activity should be avoided.

Suggested alternate Activity for plotting cooling curve with error bars (Experiment No. 14) where temperature and time are measured using a thermometer and a stop-clock (stop-watch) with complete set of data /

observations with LC of the measuring devices and $\frac{\Delta\theta}{\theta}$ and $\frac{\Delta T}{T}$ values be given.

Additionally the same curve along with error bar be asked to be drawn using two different scales and the discussion may be done using them.

ACTIVITY 4

AIM

To measure the force of limiting rolling friction for a roller (wooden block) on a horizontal plane.

APPARATUS AND MATERIAL REQUIRED

Wooden block with a hook on one side, set of weights, horizontal plane fitted with a frictionless pulley at one end, pan, spring balance, thread, spirit level, weight box and lead shots (rollers).

PRINCIPLE

Rolling friction is the least force required to make a body start rolling over a surface. Rolling friction is less than the sliding friction.

PROCEDURE

1. Check that the pulley is almost frictionless otherwise oil it to reduce friction.
2. Check the horizontal surface with a spirit level and spread a layer of lead shots on it as shown in Fig. A 4.1.
3. Weigh the wooden block.
4. Find the weight of the pan. Tie one end of the thread to the pan and let it hang over the pulley.
5. Now put the block over the layer of lead shots and tie the other end of the thread to its hook.
6. Put a small weight in the pan and observe whether the wooden block kept on rollers begin to move.

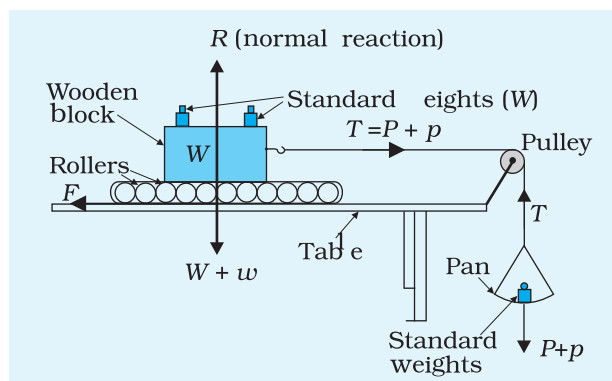


Fig. A 4.1: Setup to study rolling friction

7. If the block does not start rolling, put some more weights on the pan from the weight box increasing weights in the pan gradually till the block just starts rolling.
8. Note the total weight put in the pan, including the weight of the pan and record them in the observation table.
9. Put a 100 g weight over the wooden block and repeat Steps (7) to (9).
10. Increase the weights in steps over the wooden block and repeat Steps (7) to (9).

OBSERVATIONS

Mass of wooden block $m = \dots$ g = \dots kg

Weight of wooden block, W (mg) = \dots N

Weight on the pan

= (Mass of the pan + weight) \times acceleration due to gravity (g)

= \dots N

Table A 4.1: Table for additional weights

S. No.	Mass of standard weights on wooden block, W	Total weight being pulled = $(W + w) \times g$ = Normal Reaction, R (N)	Mass on pan (p) (kg)	Total weight (force) pulling the block and standard weights ($P+p$) g
1				
2				
3				
4				

RESULT

As the total weight being pulled increases limiting value of rolling friction increases/decreases.

PRECAUTIONS

1. The pulley should be frictionless. It should be lubricated, if necessary.
2. The portion of the string between the pulley and the hook should be horizontal.

3. The surfaces of lead shots as well as the plane and the block should be clean, dry and smooth.
4. The weights in the pan should be placed carefully and very gently.

SOURCES OF ERROR

1. Friction at the pulley tends to give larger value of limiting friction.
2. The plane may not be exactly horizontal.

DISCUSSION

1. The two segments of the thread joining the block and the pan passing over the pulley should lie in mutually perpendicular planes.
2. The total weight pulling the block (including that of pan) should be such that the system just rolls without acceleration.
3. While negotiating a curve on a road, having sand spread over it, a two wheeler has to be slowed down to avoid skidding, why?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Find the co-efficient of rolling friction μ_r by plotting the graph between rolling friction, F and normal reaction, R .
2. What will be the effect of greasing the lead shots, and the horizontal surface on which they are placed.
3. Study the rolling motion of a roller as shown in Fig. A 4.2 and compare it with the motion in the arrangement for the above Activity.

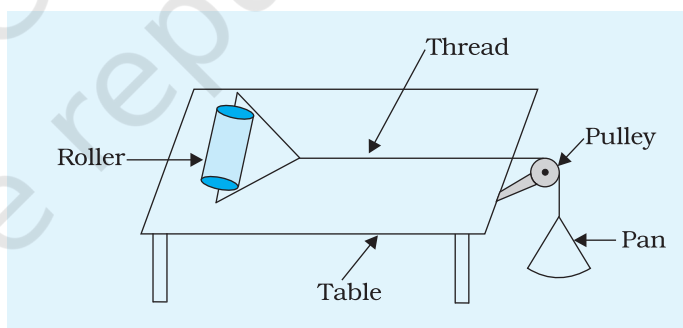


Fig. A 4.2:

ACTIVITY 5

AIM

To study the variation in the range of a jet of water with the change in the angle of projection.

APPARATUS AND MATERIAL REQUIRED

PVC or rubber pipe, a nozzle, source of water under pressure (i.e., a tap connected to an overhead water tank or water supply line), a measuring tape, large size protractor.

PRINCIPLE

The motion of water particles in a jet of water could be taken as an example of a projectile motion under acceleration due to gravity ' g '. Its range R is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

where θ_0 is the angle of projection and v_0 is the velocity of projection.

PROCEDURE

1. **Making a large protractor:** Take a circular plyboard or thick circular cardboard sheet of radius about 25 cm. Draw a diameter through its centre. Cut it along the diameter to form two dees. On one of the dees, draw angles at an interval of 15° starting with 0° .
2. Attach one end of pipe to a tap. At the other end of the pipe fix a nozzle to obtain a jet of water. Ensure that there is no leakage in the pipe.
3. Fix the protractor vertically on the ground with its graduated-face towards yourself, as shown in Fig. A 5.1.
4. Place the jet at the centre O of the protractor and direct the nozzle of the jet along 15° mark on the protractor.
5. Open the tap to obtain a jet of water. The water coming out of the

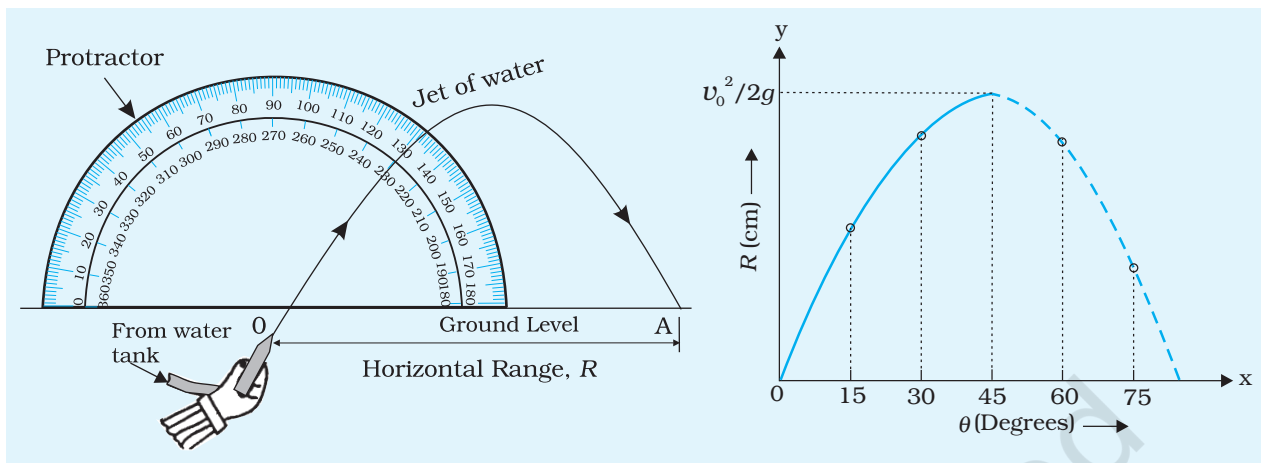


Fig. A 5.1: Setup for studying the variation in the range of a jet of water with the angle of projection

Fig. A 5.2: Variation of range with angle of projection

jet would strike the ground after completing its parabolic trajectory. Ask your friend to mark the point (A) where the water falls. Close the tap.

6. Measure the distance between point O and A. This gives the range R corresponding to the angle of projection, 15° .
7. Now, vary θ_0 in steps of 15° upto 75° and measure the corresponding range for each angle of projection.
8. Plot a graph between the angle of projection θ_0 and range R (Fig. A 5.2).

OBSERVATIONS

Least count of measuring tape = ... cm

Table A 5.1: Measurement of range

S. No.	Angle of projection θ_0 (degrees)	Range R (cm)
1	15°	
2	30°	
3	45°	
4	60°	
5	75°	

GRAPH

Plot a graph between angle of projection (on x-axis) and range (on y-axis).

RESULT

The range of jet of water varies with the angle of projection as shown in Fig. A 5. 2.

The range of jet of water is maximum when $\theta_0 = \dots^\circ$

PRECAUTIONS

1. There should not be any leakage in the pipe and the pressure with which water is released from the jet should not vary during the experiment.
2. The jet of water does not strike the ground at a point but gets spread over a small area. The centre of this area should be considered for measurement of the range.
3. The nozzle should be small so as to get a thin stream of water.

SOURCES OF ERROR

1. The pressure of water and hence the projection velocity of water may not remain constant, particularly if there is leakage in the pipe.
2. The markings on the protractor may not be accurate or uniform.

DISCUSSION

1. Why do you get same range for angles of projection 15° and 75° ?
2. Why has a big protractor been taken? Would a protractor of radius about 10 cm be preferable? Why?

SELF ASSESSMENT

1. This Activity requires the pressure of inlet water be kept constant to keep projection velocity of water constant. How can this be achieved?
2. How would the range change if the velocity of projection is increased or decreased?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Study the variation in maximum height attained by the water stream for different angles of projection.
2. Study the variation in range of water stream by varying the height at which the water supply tank is kept.
3. Take a toy gun which shoots plastic balls and repeat the Activity using this gun.
4. Calculate velocity of projection by using maximum value of horizontal range measured as above.